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An Intuitionistic Fuzzy Inventory Model using Centroid Method

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ABSTRACT

In this paper the total inventory cost and optimum order quantity are obtained in intuitionistic fuzzy sense for deteriorating items. Deterioration rate, holding cost and shortage cost are considered as vague and imprecise in nature. These parameters are considered as fuzzy parameters. Therefore in the proposed model these parameters are considered as fuzzy parameters. The vagueness of these parameters are firstly represented by triangular fuzzy numbers and then triangular intuitionistic fuzzy numbers to obtain total inventory cost. Centroid Method is used to defuzzify the fuzzy parameters to obtain total cost, optimum order quantity and shortage quantity. The model is developed in fuzzy as well as intuitionist fuzzy environment and illustrated by a numerical example to compare the results. The sensitivity analysis of the optimum solution with respect to the changes in the different parameter values is also discussed. The study concludes that intuitionistic fuzzy model is found more realistic than fuzzy and crisp model.

KEYWORDS

Intuitionistic fuzzy number (IFN), Centroid, Defuzzification, Inventory model

1. Introduction

In classical inventory model all inventory parameters are considered as deterministic. But in real life situations these parameters are vague and imprecise in nature. This causes the fluctuations in results. To tackle such type of problem, Zadeh [9] introduced the term 'Fuzzy'. Zadeh [9] treated inventory parameters as fuzzy parameters and such environment is recognised as fuzzy environment. Till now number of researchers have developed inventory models in fuzzy environment in different situations and concluded that the results are more accurate than the results of deterministic environment. These models are known as fuzzy inventory models. In fuzzy models, the fuzzy set is characterised by only membership degree. Atanassov [2] generalised the concept of fuzzy set and introduced the idea of intuitionistic fuzzy set which is characterised by membership degree non-membership degree of fuzzy parameter. Further, he proved that the results were more realistic than deterministic. As a result the 'Intuitionistic Fuzzy Set' theory has been developed and popularised to get better results. Researchers applied this the-

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ory in different areas of decision making problems including inventory control problems. Chakraborty et al. [3] gave the solution for the EOQ model using intuitionistic fuzzy optimization technique in which fuzzy parameters are transformed into corresponding interval numbers. This concept was used to minimize the interval objective function. Kaur and Mahuya [5] developed an inventory model to determine the optimal cost and an optimum order quantity of inventory by taking certain non-deterministic parameters as triangular intuitionistic fuzzy numbers. Mijanur et al. [6] proposed a more general definition of triangular intuitionistic fuzzy number (TIFN) by removing the 'normality' in the definition of intuitionistic fuzzy number with basic arithmetic operations of generalized triangular intuitionistic fuzzy numbers for first time. Varghese and Kuriakose [8] introduced a formula to find the centroid of an intuitionistic fuzzy number. Arun Prakash et al.[1] and Pardha et al.[7] ranked both triangular as well as trapezoidal intuitionistic fuzzy numbers using the centroid concept.

In this paper, fuzzy and intuitionistic fuzzy models are compared for deteriorating items using centroid method. For this, deterioration rate, holding cost and shortage cost are represented as fuzzy and intuitionistic parameters.

The solution procedure is illustrated by numerical example and the results were compared. The sensitivity analysis of the optimum solution with respect to the changes in the different parameter values is also discussed.

2. Assumptions

1. Scheduling period is constant and no lead time.
2. Demand rate is known and constant.
3. Shortages are allowed and backlogged.
4. Deteriorating rate is age specific failure rate.

3. Notations

T **:** Scheduling time.

R **:** Demand rate per unit time.

Θ **:** Deterioration rate.

$Q(t)$ **:** Inventory level at time t.

C_H **:** Holding cost.

C_1 **:** Holding cost per unit.

C_p **:** Shortage cost or penalty cost.

C_2 **:** Shortage cost per unit.

D_Θ **:** Total deteriorating units.

C_d **:** Deteriorating cost per unit.

S **:** Initial stock level.

S_1 **:** Maximum shortage level.

TC **:** Total inventory cost.

('$\sim I$' represents the intuitionistic fuzzification of the parameters)

4. Figure

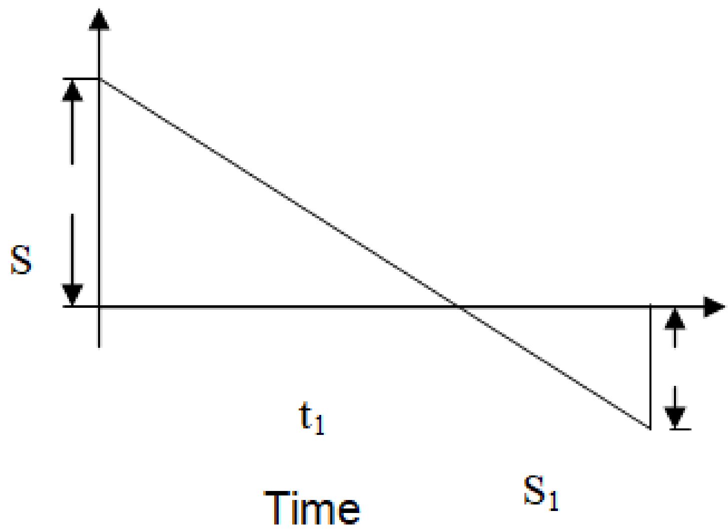


Figure 1: Intuitionistic Fuzzy Number Representation

5. Mathematical Analysis

5.1. Crisp Model

The initial stock level is S at $t = 0$, then inventory level decreases due to demand mainly and partially by deterioration. The stock reaches to zero level at $t = t_1$. Then shortages occur and accumulate to the level S_1 at $t = T$. The differential equation describing the state of inventory in the interval $(0, t_1)$ is given by,

$$\frac{d}{dt}Q(t) + \Theta Q(t) = -R \qquad ; \quad 0 \leq t \leq t_1 \tag{1}$$

Solving above differential equation using boundary condition at $t = 0$, $Q(t) = S$, we get,

$$Q(t) = -\frac{R}{\Theta} + \left(\frac{S\Theta + R}{\Theta}\right) e^{-\Theta t} \qquad ; \quad 0 \leq t \leq t_1 \tag{2}$$

Using boundary condition at $t = t_1$, $Q(t_1) = 0$, we get

$$t_1 = \frac{1}{\Theta} \log \left(1 + \frac{S\Theta}{R}\right) \tag{3}$$

The differential equation describing the state of inventory in the interval (t_1, T) is given by,

$$\frac{d}{dt} Q(t) = -R \qquad ; \quad t_1 \leq t \leq T \tag{4}$$

Integrating both sides and solving using condition at $t = t_1$, $Q(t_1) = 0$, we get,

$$Q(t) = -Rt + Rt_1 \qquad ; \quad t_1 \leq t \leq T \tag{5}$$

Using condition at $t = T$, $Q(t) = -S_1$, we get,

$$S_1 = RT - R \cdot \frac{1}{\Theta} \log \left(1 + \frac{S\Theta}{R}\right) \tag{6}$$

Total deteriorating units during the time interval $(0, T)$ are

$$D_\Theta = \int_0^{t_1} \Theta Q(t)\, dt \qquad ; \quad 0 \leq t \leq t_1$$

Solving above integral using equation (2), we get,

$$D_\Theta = -Rt_1 - \left[\frac{S\Theta + R}{\Theta}(e^{-\Theta t_1} - 1)\right]$$

Therefore the deteriorating cost is given by,

$$C_D = C_d \cdot D_\Theta$$

$$\therefore \quad C_D = C_d \left[-Rt_1 - \left(\frac{S\Theta + R}{\Theta}(e^{-\Theta t_1} - 1)\right)\right] \tag{7}$$

Holding cost over the time period $(0, T)$ is given by,

$$C_H = C_1 \int_0^{t_1} Q(t)\, dt$$

Solving above integral using equation (2), we get,

$$C_H = C_1 \left[-\frac{Rt_1}{\Theta} - \left(\frac{S\Theta + R}{\Theta^2} (e^{-\Theta t_1} - 1) \right) \right] \tag{8}$$

Shortage cost is given by

$$C_P = C_2 \left[- \int_{t_1}^{T} Q(t)\, dt \right]$$

Solving above integral by using equation (5), we get

$$C_P = C_2 \left[\frac{R}{2} (T - t_1)^2 \right] \tag{9}$$

Then the total inventory cost is given by,

$$TC = C_H + C_D + C_P$$

$$TC = C_1 \left[-\frac{Rt_1}{\Theta} - \left(\frac{S\Theta + R}{\Theta^2} \right) (e^{-\Theta t_1} - 1) \right] + C_d \left[-Rt_1 - \left(\frac{S\Theta + R}{\Theta} \right) (e^{-\Theta t_1} - 1) \right] + C_2 \left[\frac{R}{2} (T - t_1)^2 \right] \tag{10}$$

The above equation can be simplified using series form of logarithmic term and ignoring second and higher terms as follows

$$TC = C_1 \frac{S^2}{R} + C_d \Theta \frac{S^2}{R} + C_2 \left[\frac{R}{2} \left(T - \frac{S}{R} \right)^2 \right] \tag{11}$$

to obtain optimum order quantity differentiating TIC partially w.r.t. S and equate to zero

$$\frac{dTC}{dS} = C_1 \left(\frac{S}{R + S\Theta} \right) + C_d \left(\frac{S\Theta}{R + S\Theta} \right) - C_2 \left(\frac{R}{R + S\Theta} \right) \left[T - \frac{1}{\Theta} \log \left(1 + \frac{S\Theta}{R} \right) \right] \tag{12}$$

By ignoring Second and higher terms we get
The optimum order level is given by,

$$S^o = \frac{C_2 RT}{C_1 + C_d \Theta + C_2} \tag{13}$$

5.2. Fuzzy Model

In the above developed crisp model, it is assumed that all the inventory parameters are fixed or could be predicted with certainty, but in real life situations, these parameters will fluctuate little from the actual value. Therefore such parameters could not be assumed to be constant. Hence as per Zadeh (1965)[9], these parameters are treated as fuzzy parameters. Usually rate of deterioration, holding cost and shortage cost are vague in nature, thus instead of considering them as constant the proposed model is developed with the assumption that these parameters are fuzzy parameters and represented by fuzzy numbers.

Hence the fuzzy total inventory cost is

$$T\tilde{C} = \tilde{C}_1 \otimes \left(\frac{\tilde{S}^2}{R}\right) + C_d \otimes \tilde{\Theta} \otimes \left(\frac{\tilde{S}^2}{R}\right) + \tilde{C}_2 \otimes \left[\frac{R}{2}\left(T - \frac{\tilde{S}}{R}\right)^2\right]$$

$$\tilde{S} = \frac{\tilde{C}_2 \otimes RT}{\tilde{C}_1 \oplus \tilde{C}_d \otimes \tilde{\Theta} \oplus \tilde{C}_2}$$

where

$$\frac{\tilde{S}^2}{R}, \quad \tilde{C}_2 \otimes RT, \quad C_d \otimes \tilde{\Theta} \quad \text{are fuzzy points.}$$

5.2..1 Defuzzification by Centroid of Fuzzy numbers

Here deterioration rate, holding cost and shortage cost are represented by triangular fuzzy numbers. By Centroid method fuzzy parameters are defuzzified. The Centroid of

$$\tilde{A} = (a, b, c) \text{ can be generally defined by } C(\tilde{A}) = \frac{1}{3}(b + a + c)$$

$$= b + \frac{1}{3}(\Delta_2 - \Delta_1) \quad \text{where} \quad a = b - \Delta_1 \text{ and } c = b + \Delta_2 \text{ and } 0 < \Delta_1, \Delta_2$$

5.3. Intuitionistic Fuzzy Model

Generally vague and imprecise parameters are treated as fuzzy parameters. However, Atanassov [2] treated such parameters as Intuitionistic fuzzy parameters in an appropriate way to get more accurate results.

In this model deterioration rate, holding cost and shortage cost are considered as intuitionistic fuzzy parameters and represented by triangular intuitionistic fuzzy numbers. Then the Intuitionistic fuzzy total inventory cost is

$$T\tilde{C}^I = \tilde{C}_1^I \otimes \left(\frac{(\tilde{S}^I)^2}{R}\right) + C_d \otimes \tilde{\Theta}^I \left(\frac{(\tilde{S}^I)^2}{R}\right) + \tilde{C}_2^I \otimes \left[\frac{R}{2}\left(T - \frac{\tilde{S}^I}{R}\right)^2\right]$$

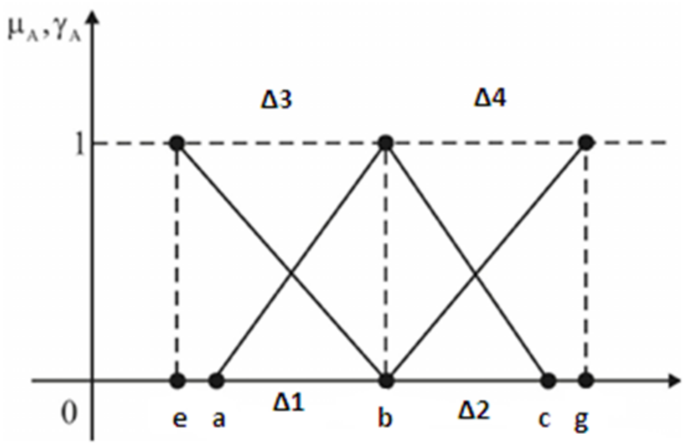
where

$$\tilde{S}^I = \frac{\tilde{C}_2^I \otimes RT}{\tilde{C}_1^I \oplus \tilde{C}_d^I \otimes \tilde{\Theta}^I \oplus \tilde{C}_2^I}$$

5.3..1 Defuzzification by Centroid of Intuitionistic Fuzzy Numbers

Here deterioration rate, holding cost and shortage cost are represented by triangular Intuitionistic Fuzzy numbers. By Centroid method Intuitionistic Fuzzy parameters are defuzzified.

Def:If A is any Intuitionistic Fuzzy Number then it is shown in figure as follows:



Then the Centroid of TIFN $A\{(a, b, c); (e, b, g)\}$ is

$$C(\tilde{A}^I) = \frac{1}{3}\left[\frac{(g-e)(b-2g-2e)+(c-a)(a+b+c)+3(g^2-e^2)}{(g-e+c-a)}\right]$$

Let $\Delta_1 = b-a, \quad \Delta_2 = c-b, \quad \Delta_3 = b-e, \quad \Delta_4 = g-b$

Then Centroid of TIFN $A\{(a, b, c); (e, b, g)\}$ becomes

$$C(\tilde{A}^I) = A + \frac{1}{3}\left[\frac{(\Delta_2^2-\Delta_1^2)+(\Delta_4^2-\Delta_3^2)}{(\Delta_1+\Delta_2+\Delta_3+\Delta_4)}\right]$$

Generally, In real life, it is very difficult to consider the deterioration rate θ over a total time period T. Then it is easy to locate the deterioration rate in an interval

$\{(\theta-\Delta_1, \theta, \theta+\Delta_2); (\theta-\Delta_3, \theta, \theta+\Delta_4)\}$ where $0 < \Delta_1 < \theta$ and $0 < \Delta_1, \Delta_2$ and $\Delta_1 \neq \Delta_2$

will be decided by decision maker and θ is a known number.

Then the Centroid of $\tilde{\theta}$ is

$$C(\tilde{\theta}^I) = \theta + \frac{1}{3}\left[\frac{(\Delta_2^2-\Delta_1^2)+(\Delta_4^2-\Delta_3^2)}{\Delta_1+\Delta_2+\Delta_3+\Delta_4}\right]$$

Where $\Delta_1 = \theta - a,\ \Delta_2 = c - \theta,\ \Delta_3 = \theta - e,\ \Delta_4 = g - \theta$

Similarly Centroids of $\tilde{C_1}^I$ and $\tilde{C_2}^I$ are

$$C(\tilde{C_1}^I) = C_1 + \frac{1}{3}\left[\frac{(\Delta_2^2 - \Delta_1^2) + (\Delta_4^2 - \Delta_3^2)}{\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4}\right]$$

Where $\Delta_1 = C_1 - a,\ \Delta_2 = c - C_1,\ \Delta_3 = C_1 - e,\ \Delta_4 = g - C_1$

$$C(\tilde{C_2}^I) = C_2 + \frac{1}{3}\left[\frac{(\Delta_2^2 - \Delta_1^2) + (\Delta_4^2 - \Delta_3^2)}{\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4}\right]$$

Where $\Delta_1 = C_2 - a,\ \Delta_2 = c - C_2,\ \Delta_3 = C_2 - e,\ \Delta_4 = g - C_2$

6. Numerical Example

6.1. Crisp Model

Input values:

$$R = 100, \quad C_1 = 0.4, \quad C_2 = 1.1, \quad C_d = 0.3, \quad T = 1, \quad \theta = 0.35$$

Output values:

$$S = 68.54, \quad t_1 = 0.61, \quad S_1 = 38.57, \quad TC = 18.43$$

6.2. Defuzzification by Centroid

Table 1: Comparison of Centroid Values of FUZZY and IFN for C_1

Case	Δ_1	Δ_2	Δ_3	Δ_4	**Centroid(FUZZY C_1)**	**Centroid(IFN C_1)**
1	0.01	0.1	0.05	0.13	0.4000000	0.427931
2	0.05	0.1	0.1	0.2	0.4166667	0.427778
3	0.1	0.1	0.12	0.25	0.4333333	0.428129
4	0.1	0.15	0.15	0.18	0.4500000	0.412874
5	0.1	0.15	0.2	0.15	0.3833333	0.397222
6	0.15	0.15	0.15	0.2	0.3666667	0.408974
7	0.2	0.15	0.2	0.25	0.3833333	0.402083
8	0.2	0.05	0.22	0.1	0.3500000	0.355614
9	0.2	0.01	0.25	0.03	0.3666667	0.330952
10	0.18	0.2	0.2	0.25	0.3833333	0.412088

(IFN : Intuitionistic Fuzzy Number)

Table 2: Variation in the Values of C_2 with Centroid Comparison of FUZZY and IFN

Case	Δ_1	Δ_2	Δ_3	Δ_4	**Centroid(FUZZY** C_2**)**	**Centroid(IFN** C_2**)**
1	0.1	0.1	0.1	0.13	1.100000	1.105349
2	0.1	0.1	0.15	0.2	1.133333	1.110606
3	0.15	0.1	0.2	0.25	1.066667	1.104762
4	0.15	0.15	0.25	0.18	1.083333	1.086256
5	0.15	0.15	0.3	0.15	1.133333	1.070000
6	0.2	0.15	0.2	0.2	1.133333	1.092222
7	0.3	0.15	0.5	0.25	1.066667	1.029167
8	0.4	0.05	0.5	0.1	1.066667	0.973810
9	0.5	0.01	0.55	0.03	1.083333	0.931346
10	0.45	0.4	0.5	0.45	1.116667	1.083333

Table 3: Variation in the Values of θ with Centroid Comparison of FUZZY and IFN

Case	Δ_1	Δ_2	Δ_3	Δ_4	**Centroid(FUZZY** θ**)**	**Centroid(IFN** θ**)**
1	0.05	0.01	0.06	0.06	0.350000	0.345556
2	0.05	0.01	0.09	0.07	0.383333	0.341515
3	0.08	0.01	0.05	0.10	0.316667	0.351667
4	0.08	0.015	0.10	0.10	0.333333	0.343023
5	0.10	0.01	0.15	0.20	0.383333	0.355507
6	0.10	0.10	0.20	0.10	0.316667	0.330000
7	0.10	0.10	0.10	0.15	0.316667	0.359259
8	0.15	0.02	0.18	0.03	0.383333	0.302982
9	0.15	0.10	0.20	0.15	0.333333	0.333333
10	0.015	0.10	0.03	0.18	0.333333	0.392333

6.2..1 Fuzzy Model

Table 4: Fuzzy Total Cost (for $R = 100, T = 1$and $C_d = 0.3$)

Case No.	C_1	C_2	θ	S	TC	t_1	S_1
1	0.400000	1.100000	0.350000	68.53583	29.16562	0.614317	38.56832
2	0.416667	1.133333	0.383333	68.06807	30.41152	0.604820	39.51801
3	0.433333	1.066667	0.316667	66.87565	29.48078	0.606583	39.34174
4	0.450000	1.083333	0.333333	66.32653	30.33762	0.599228	40.07721
5	0.383333	1.133333	0.383333	69.45863	29.32783	0.615825	38.41752
6	0.366667	1.133333	0.316667	71.05538	28.05642	0.640888	35.91117
7	0.383333	1.066667	0.316667	69.03991	27.91195	0.624393	37.56074
8	0.350000	1.066667	0.383333	69.64091	27.46743	0.617264	38.27361
9	0.366667	1.083333	0.333333	69.89247	27.70648	0.628290	37.17104
10	0.383333	1.116667	0.333333	69.79167	28.63760	0.627472	37.25281

6.2..2 Intuitionistic Fuzzy Model

Table 5: Intuitionistic Fuzzy Total Cost (for $R = 100, T = 1$and $C_d = 0.3$)

Case	C_1	C_2	θ	S	TC	t_1	S_1
1	0.427931	1.105349	0.345556	67.52504	30.06753	0.606916	39.30842
2	0.427778	1.110606	0.341515	67.68528	30.09022	0.608913	39.10866
3	0.428129	1.104762	0.351667	67.42970	30.12263	0.605097	39.49031
4	0.412874	1.086256	0.343023	67.80470	29.34261	0.609621	39.03785
5	0.397222	1.070000	0.355507	67.98509	28.77244	0.608918	39.10816
6	0.408974	1.092222	0.330000	68.25550	29.16881	0.615574	38.44263
7	0.402083	1.029167	0.359259	66.87122	28.44740	0.599305	40.06953
8	0.355614	0.973810	0.302982	68.56277	25.80180	0.622957	37.70426
9	0.330952	0.931346	0.333333	68.36578	24.80229	0.615882	38.41182
10	0.412088	1.083333	0.392333	67.15757	29.73675	0.596103	40.38971

7. Concluding Remark

In Table 1, Table 2 and Table 3 variation in the values of fuzzy and IFN (intuitionistic fuzzy number) of C1, C2 and C3 are obtained. Using these values sensitivity analysis for optimum values of total cost, optimum order quantity and shortage quantity of Fuzzy and Intuitionistic Fuzzy model are presented in Table 4 and 5 respectively, for various values of Deterioration rate, holding cost and shortage cost. The optimum values of both models are nearly equal but values in Intuitionistic Fuzzy model seem to be more realistic. Decision maker can select the optimum results of any one suitable case. From the above results it can be concluded that the optimum values of intuitionistic fuzzy model are more accurate than crisp and fuzzy model.

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